The Structure of Nuclei

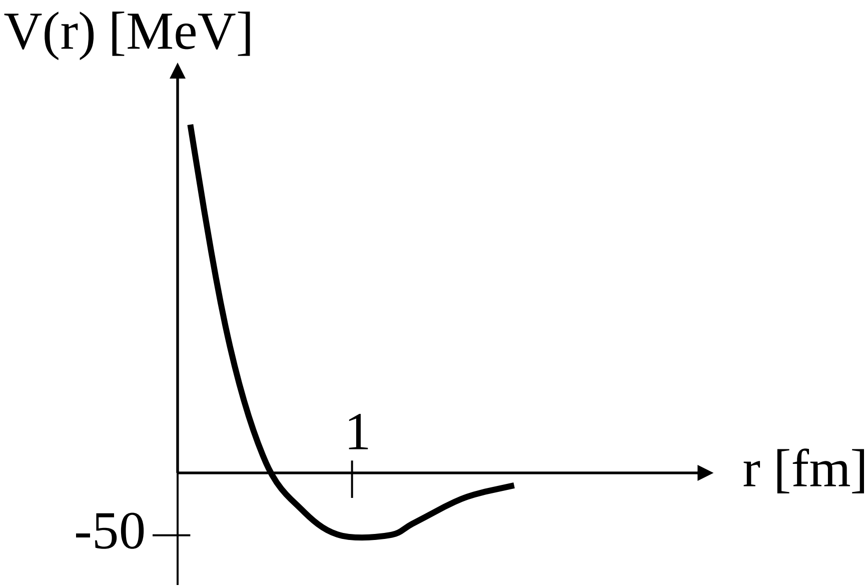
Wolfgang Bentz

Department of Physics, School of Science

Tokai University, Japan

1) Nucleon-Nucleon (NN) force and light nuclei

From the study of NN-scattering and nuclear binding energies, the following behavior of the NN-potential emerges:



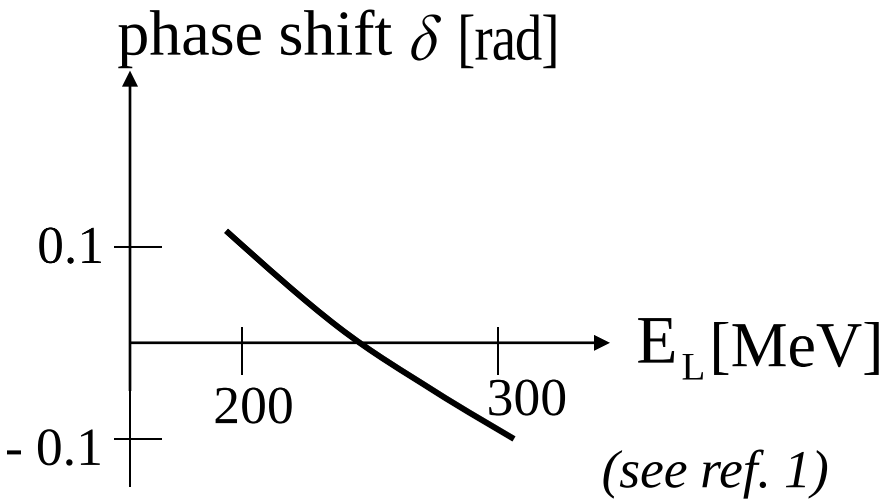
i) ***Short range repulsion*** ($r \leq 0.5$ fm)

ii) ***medium range attraction*** ($r \approx 1$ fm)

iii) ***exponential tail*** $\propto \exp(-cr)$ ($r \approx 2$ fm)

Experimental information:

i) **<u>Short range repulsion</u>**: The *phase shift* for NN scattering (for L=S=0) changes sign at $E_L \approx 250$ MeV



(see ref. 1)

Laboratory energy: E_L

Center of mass energy: $\frac{1}{2} E_L \equiv \frac{k^2}{2\,m}$

(m=M/2=reduced mass ;

$\hbar\, k =$ momentum in c.m.)

$\delta < 0$ means ***repulsion***. Behavior for high energies is similar to scattering on hard sphere (radius r_C): $\delta(k) \approx -kr_C$, ($r_C \approx 0.5$ fm)

<u>ii) Intermediate range attraction</u>:

<u>*Range:*</u> The binding energy per nucleon (B/A) increases rapidly up to A=4 (^{4}He) $\Rightarrow$ In ^{4}He , all nucleons are within the range of their attraction $\Rightarrow$ The range of the nuclear force (R) is about the same as the radius of ^{4}He

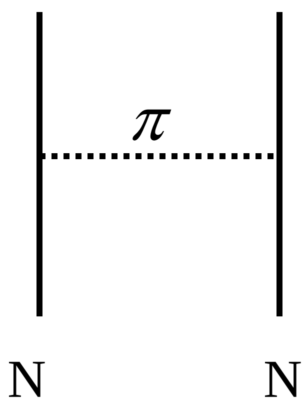
$\Rightarrow$ $R \approx 1.5$ fm.

Strength of attraction: The deuteron (p+n) is a very weakly bound state and has no excited states. If we use a square well potential for the attraction, we obtain from the Schroedinger equation:

$$\frac{2mV_0R^2}{\hbar^2} \approx \frac{\pi^2}{4} \rightarrow V_0 \approx 45 \text{ MeV}$$

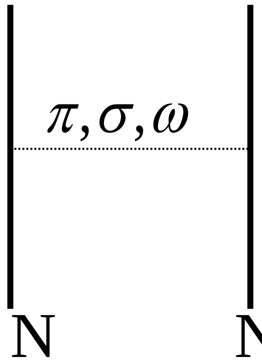
(Here we used m=M/2=reduced mass, and R=1.5 fm.)

iii) ***Long range tail*** of NN potential is described by the **pion exchange potential** ($\Rightarrow$ *Yukawa theory*)

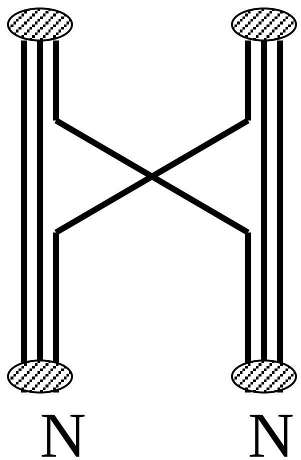


$$\propto \frac{1}{r}\, e^{-m_\pi r\, c/\hbar} \qquad (m_\pi\text{=pion mass})$$

Theoretical description of NN potential
by meson and quark exchange between nucleons:

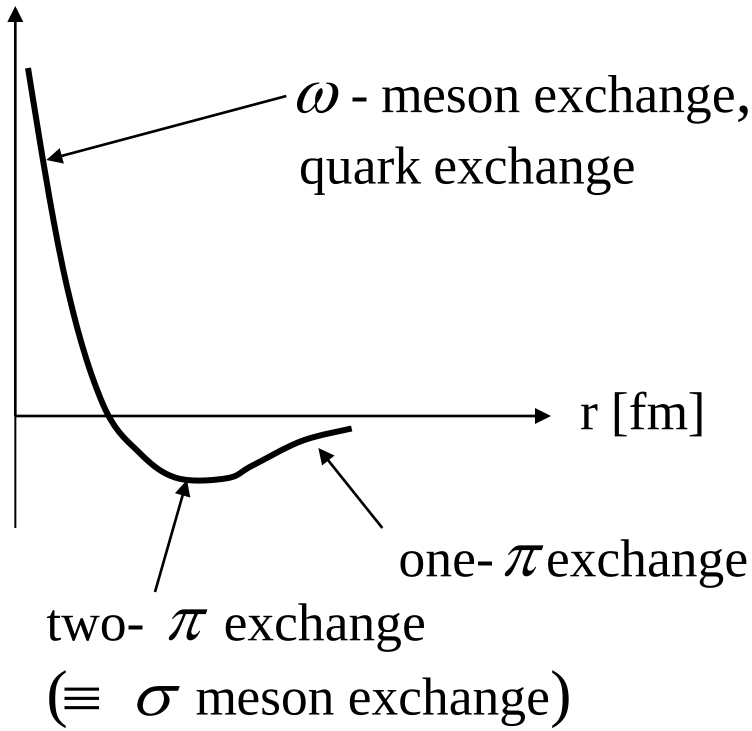


(meson exchange)



(quark exchange)

(see ref. 2)

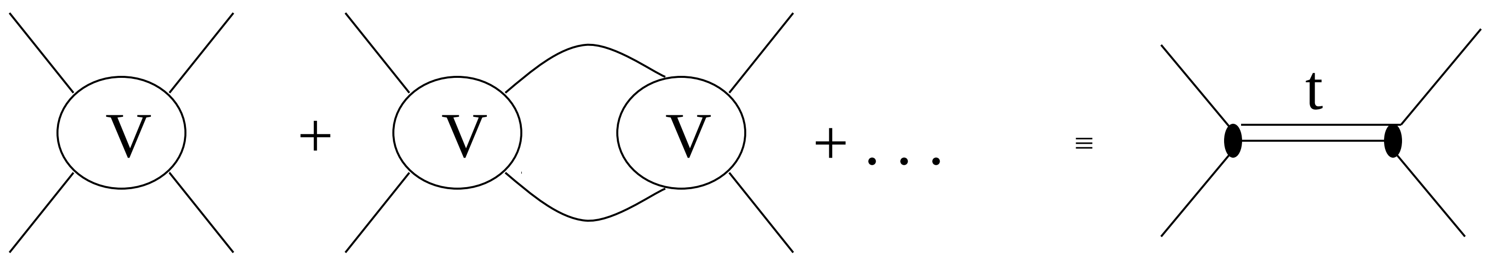


meson	mass [MeV]
π	140
σ	$\approx$ 550
ω	_783

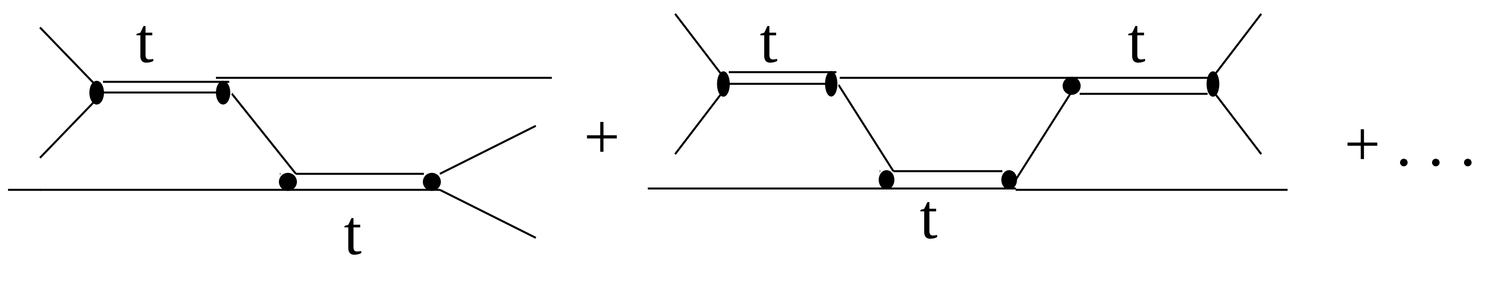
Light nuclei (A=2,3,4)

Systems with A=2,3,4 nucleons can be treated **exactly** for a given NN potential. One solves the following equations for the scattering amplitudes (t-matrices), and looks for bound state poles:

A=2: **Bethe-Salpeter equation**



A=3: **Faddeev equation**

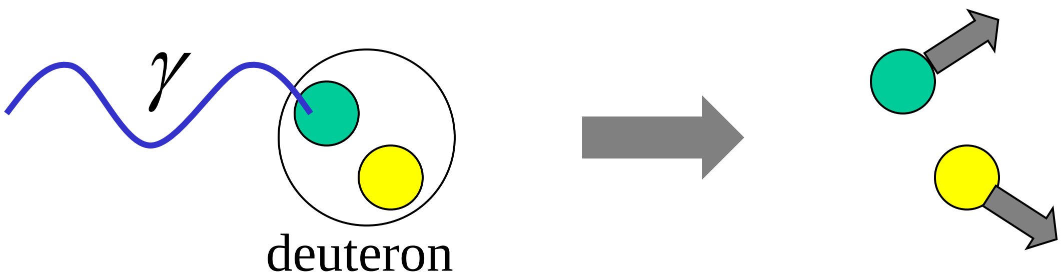


This method is equivalent to solving the Schroedinger equation for A=2,3 nucleons. *(see ref. 3)*

Examples for recent research on A=2,3 nucleon systems:

a) *Photodisintegration of the deuteron at high energies*:

$$\gamma + d \longrightarrow n + p$$



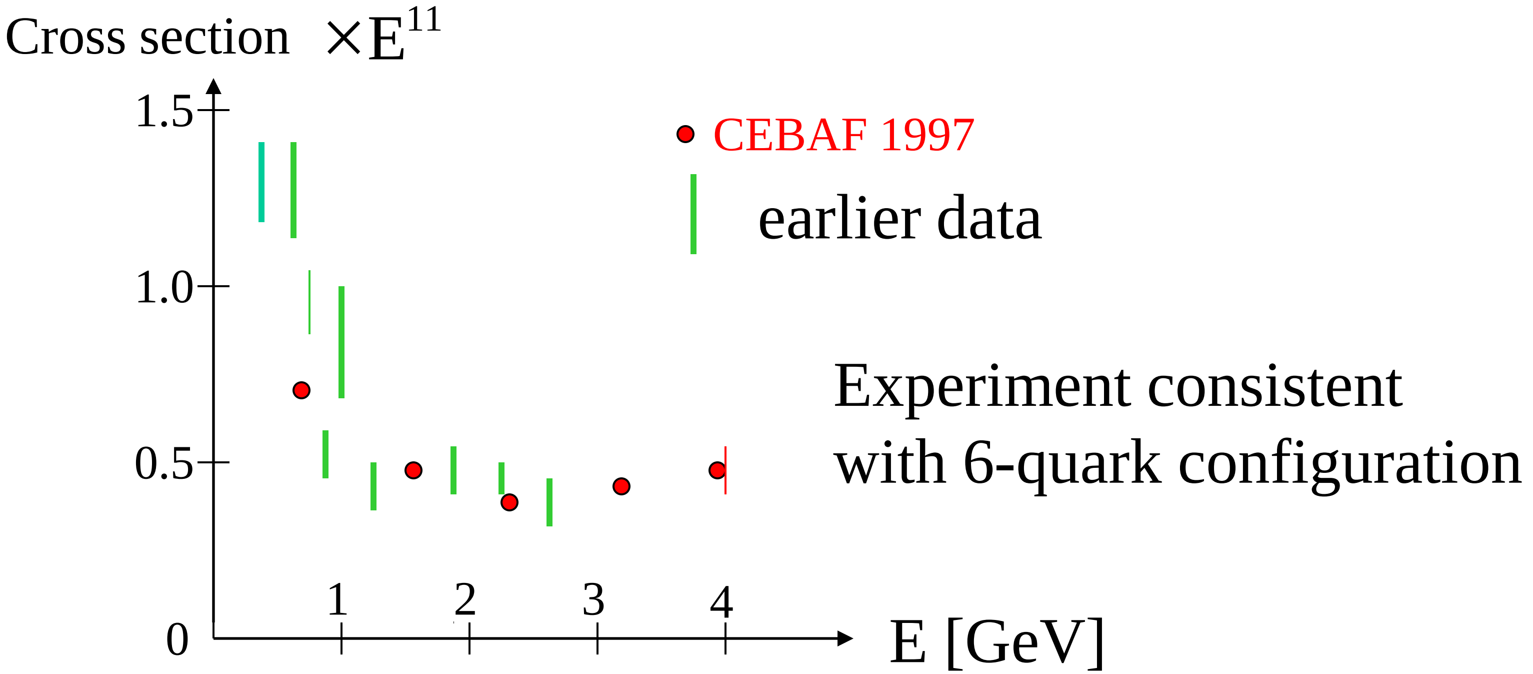
(see ref. 4)

The photon with energy E=pc acts as a "**microscope**" with wave length

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar c}{E} = \frac{1.2}{E[\mathrm{GeV}]}\ [\mathrm{fm}]$$

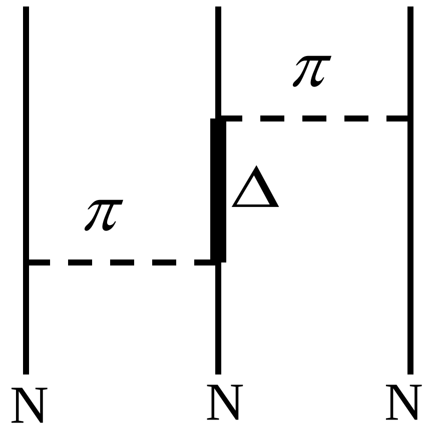
For small E, the photon can "see" only 2 separate nucleons, but for high E it could see also the configuration with 2 overlapping nucleons (6 quark configuration).

Theory predicts the energy dependence of the 6-quark process as $1/E^{11}$ ⇨ The cross section times E^{11} should be a constant for large E?



(from ref. 5, p. 57)

b) *Evidence for 3-body force in three nucleon systems*:



2π exchange with intermediate Δ excitation gives attraction.

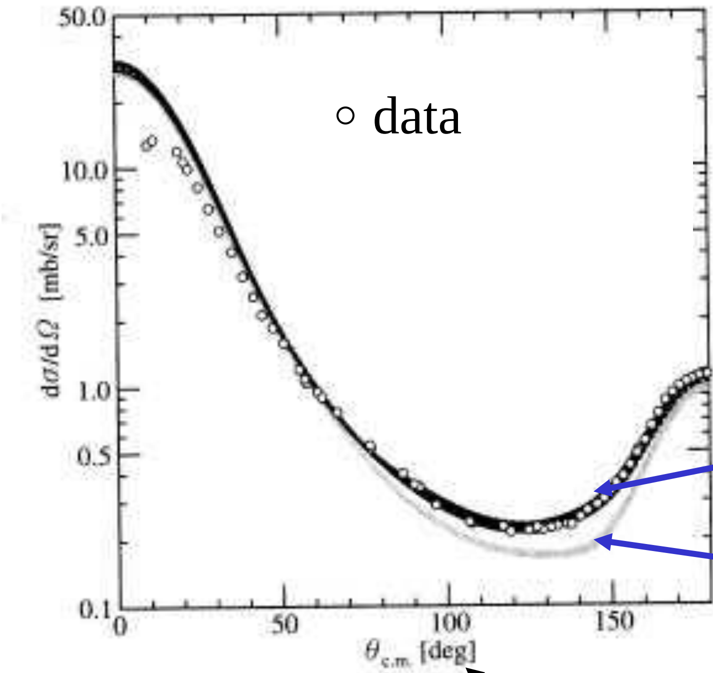
(Note: The Δ is an excited state of the nucleon with mass 1232 MeV.)

2-body forces (adjusted to A=2 data) alone give <u>too small binding energy</u> for ^{3}H : From Faddeev calculations,

2-body force	E(2-body)	E(2-body+3-body)	exp.
Bonn	7.953 MeV	8.483 MeV	8.482 MeV
Nimjegen	7.664 MeV	8.480 MeV	8.482 MeV

(see ref. 6)

Recent experiment at RIKEN, Japan:



Differential cross section for **elastic scattering of deuterons** (270 MeV kinetic energy) **on protons** (hydrogen),and comparison with Faddeev calculations.

2-body+3-body force

2-body force only

(from ref. 7)

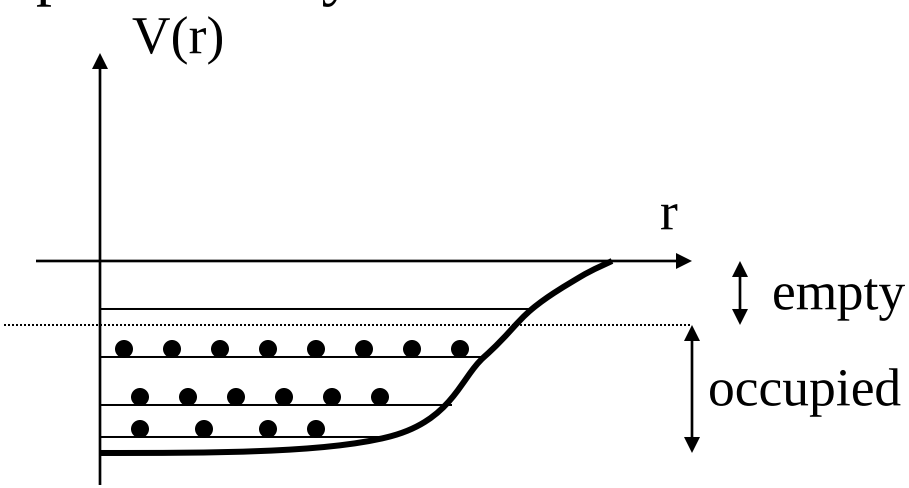
scattering angle in c.m. system

Experimental data support the presence of an attractive 3-body force.

2) Heavy nuclei

There are 2 extreme *approximation schemes* to describe heavy nuclei: (a) Single particle motion, and (b) Collective motion.

(a) **Single particle motion**: One assumes that all nucleons move independently in an average potential, which is produced by themselves.

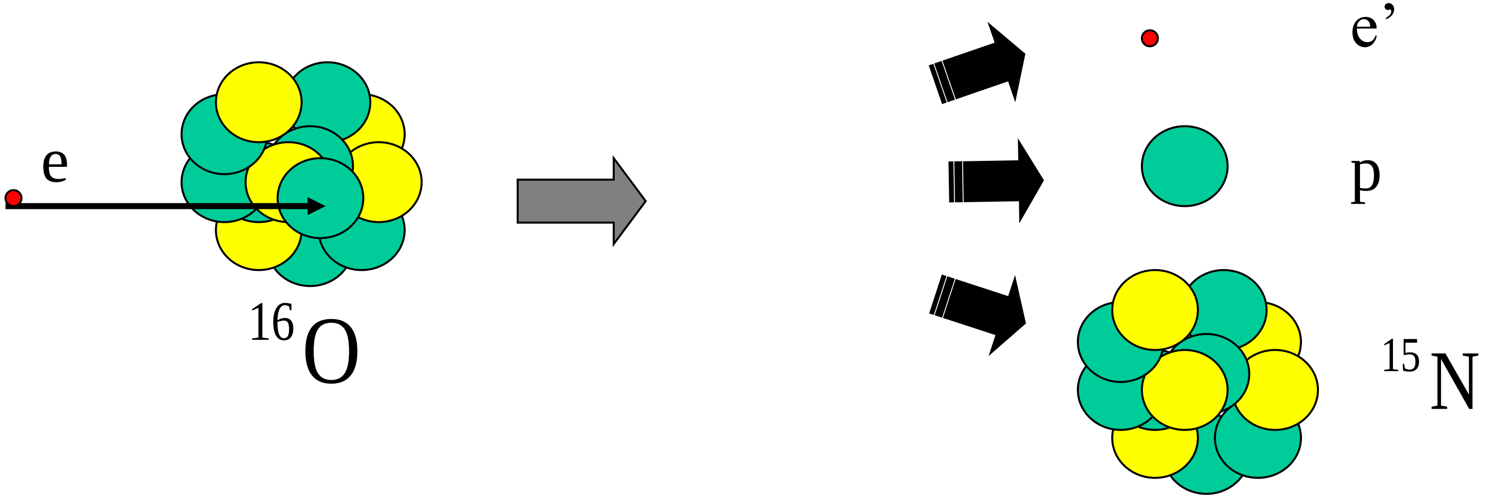


This leads to the **NUCLEAR SHELL MODEL** (M.G. Mayer, J.H.D. Jensen, 1948). (*see ref. 1*)

There are many **evidences for the nuclear shell structure**:

Recent experiments: **Proton knock-out processes**, for example

$$^{16}\mathrm{O} + \mathrm{e} \rightarrow {}^{15}\mathrm{N} + \mathrm{e'} + \mathrm{p}$$



In the experiment, the momenta of e' and p are measured.
The "***missing energy***"

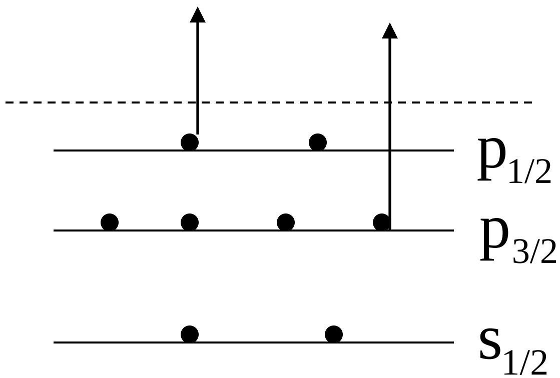
$$E_m = E_e - (E_{e'} + E_p + E_{kin}[^{15}\mathrm{N}])$$

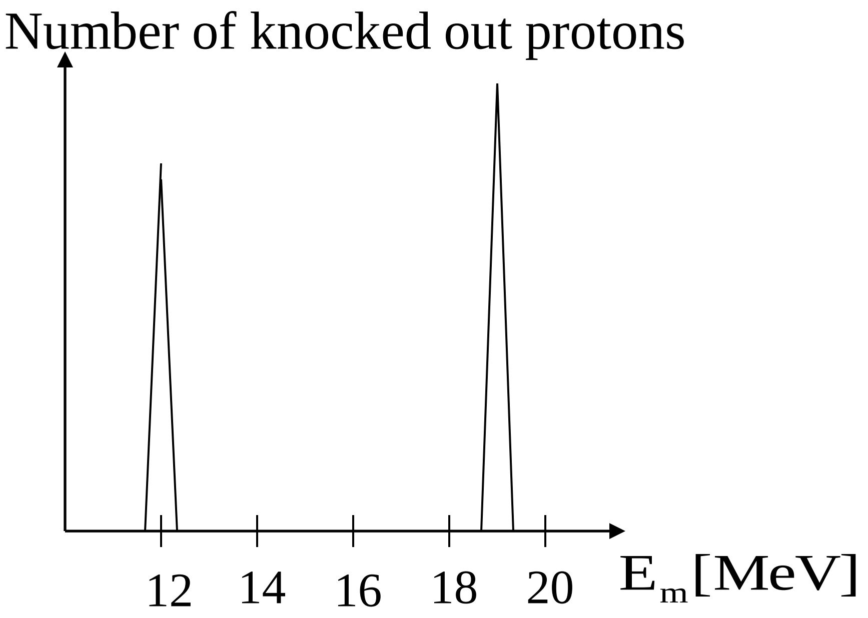
is the energy necessary to knock out the proton from $^{16}\mathrm{O}$.

Theoretical picture:

Experimental observation:

Shell structure (protons in ^{16}O)





(see ref. 8; ref. 5, p. 53)

Experimental data on knock-out processes provide evidence for the nuclear shell structure.

b) Nuclear collective motion: Many nucleons in the nucleus move together coherently.

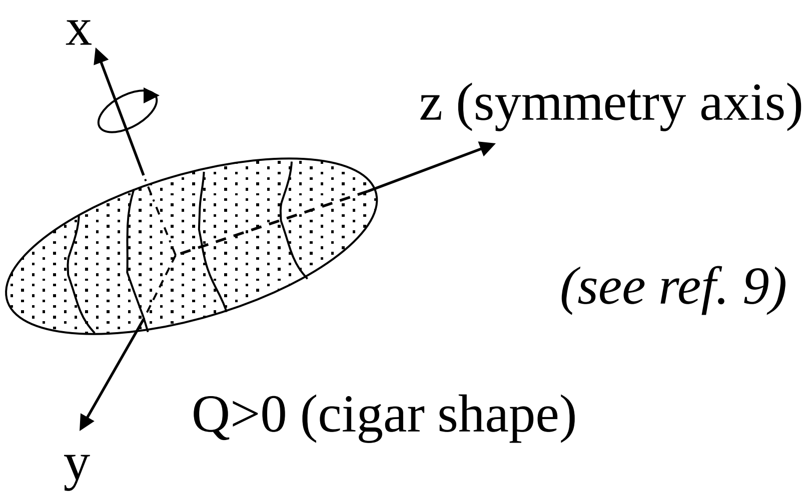
i) **Nuclear deformations**:

Some nuclei have intrinsic deformations, i.e., their ***quadrupole moments***

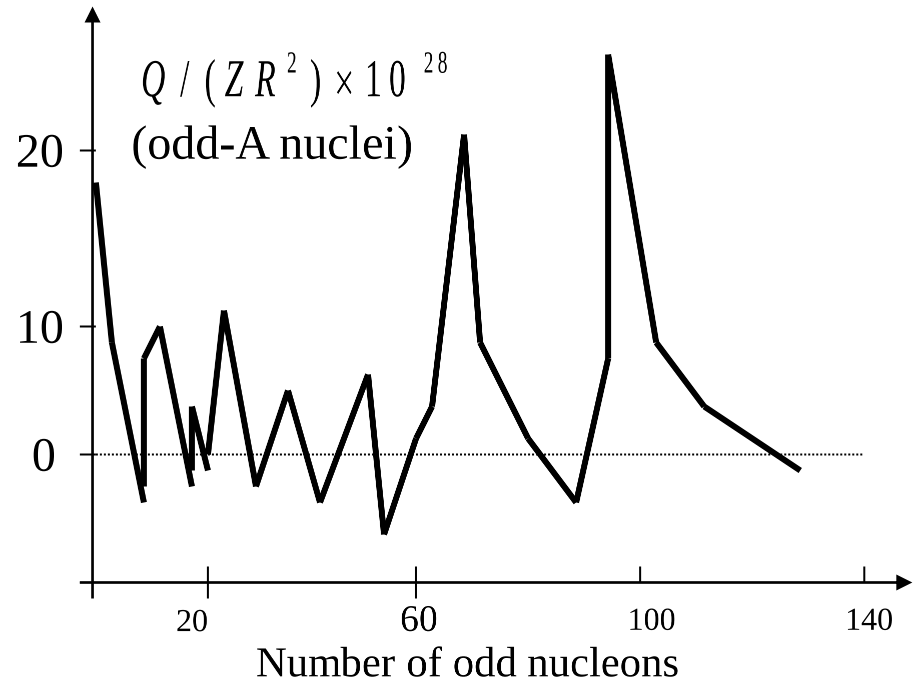
$$Q = Z\int d^3r\,\rho(x,y,z)\,(3z^2 - r^2)$$

are nonzero.

In most cases Q>0, i.e.., the nuclei have “cigar shape”:



(see ref. 9)



If a deformed nucleus would be fixed in space, its existence would break rotational symmetry (and angular momentum conservation), although the Hamiltonian is rotational invariant. **The only way to have deformation and, at the same time, good angular momentum, is that <u>the deformed nucleus *rotates*</u>.**

In quantum mechanics, **rotation around a symmetry axis makes no sense**: $R_3 \psi = -i\hbar\, \partial\psi/\partial\phi = 0$ if 3 is the symmetry axis, and ϕ the angle in the 1,2 plane. However, rotation around the 1, 2 axes can lead to observable effects.

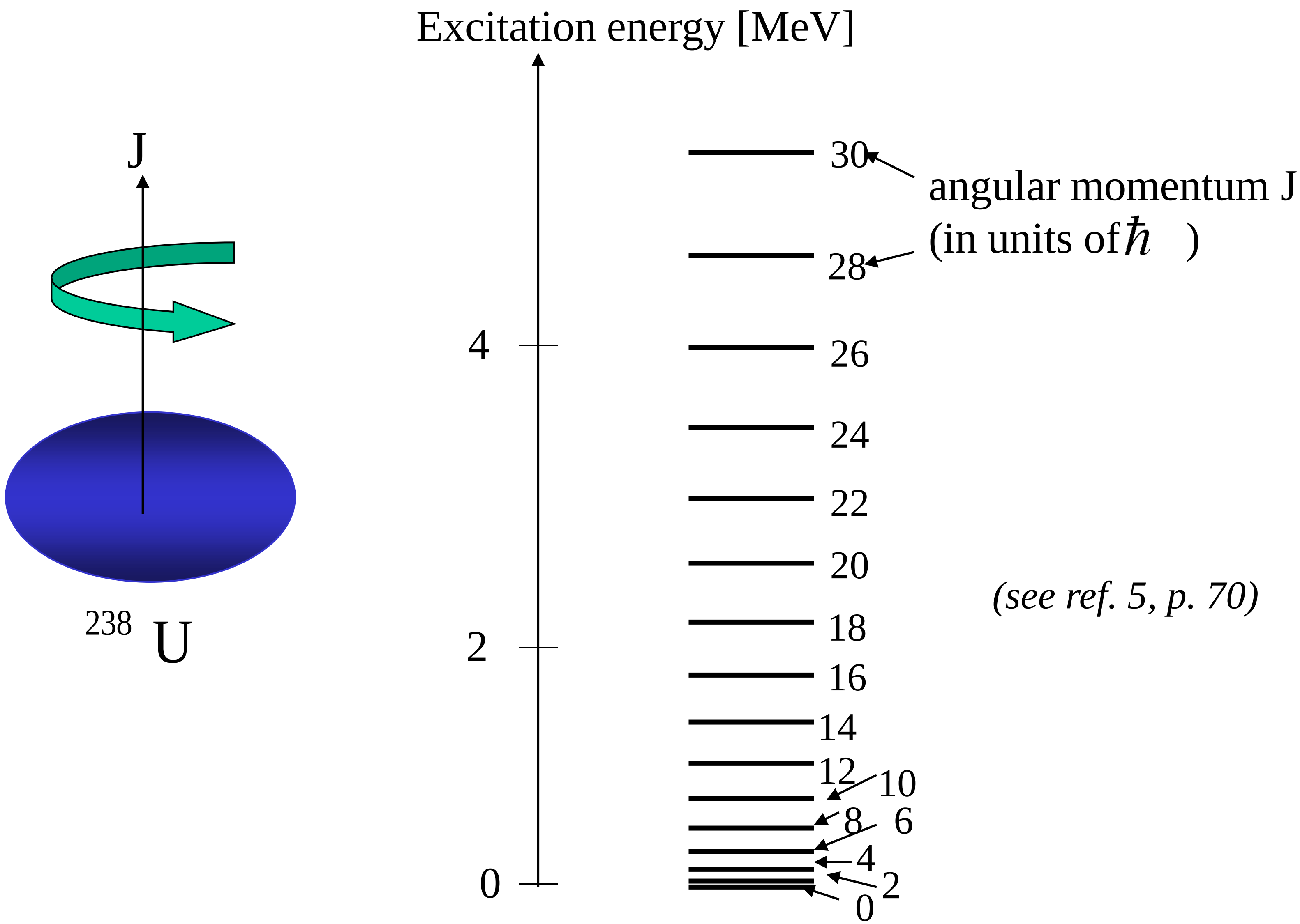
ii) **<u>Nuclear rotations</u>**:

Many deformed nuclei show an excitation spectrum of a **rigid rotor:**

$$E_J = \frac{\hbar^2}{2I}\, J\,(J+1) \qquad (J=0,2,4,\ldots)$$

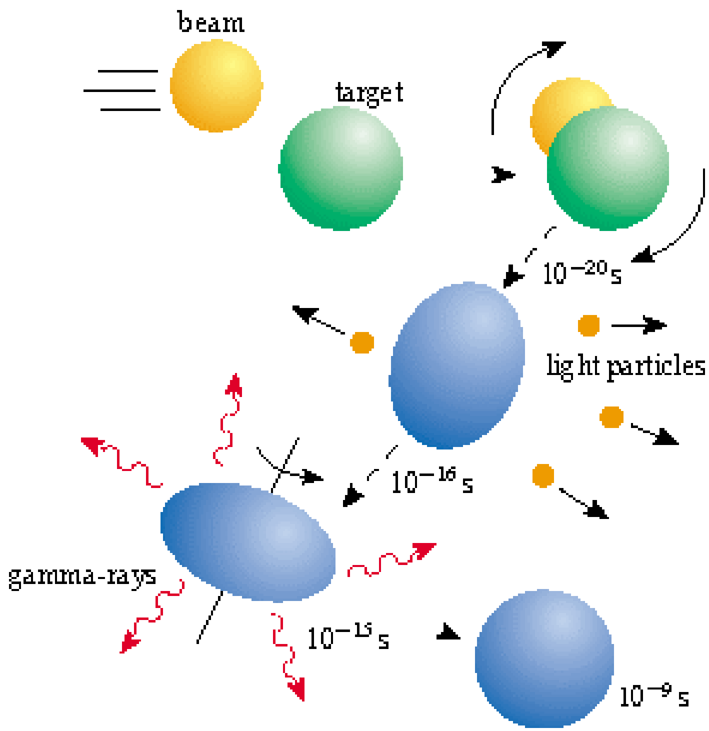
I = **moment of inertia.** (For a symmetric shape, the rotational wave function Y_{JM} should not change sign under reflection in the 1,2 plane $\longrightarrow$ <u>J is even.</u>)

Example: Excitation spectrum of ^{238}U

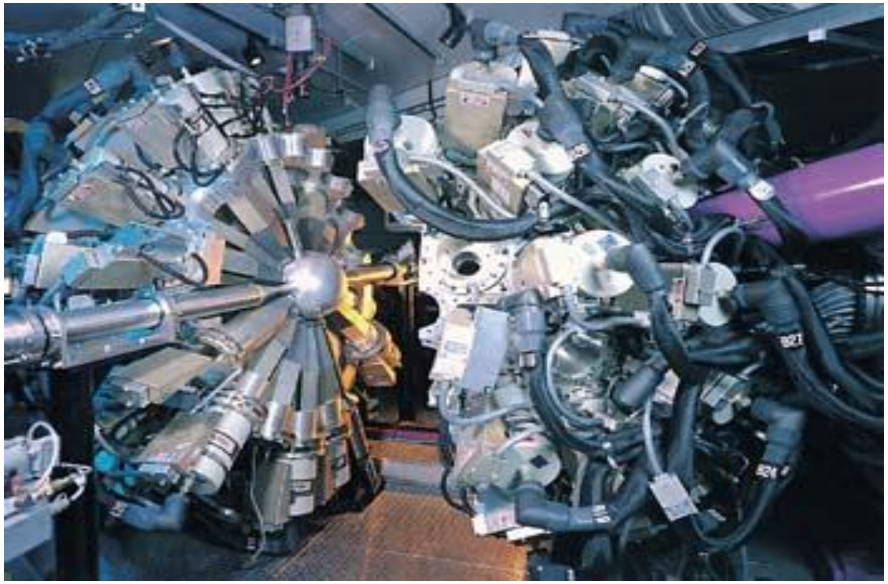


The study of **<u>rotating nuclei with large deformations</u>** is a topic of recent experimental and theoretical research.

Such nuclei are produced as **compound systems in fusion reactions**. After evaporation of particles (nucleons, alpha particles), the compound nucleus emits γ rays, whose energies are measures with 4π detectors:

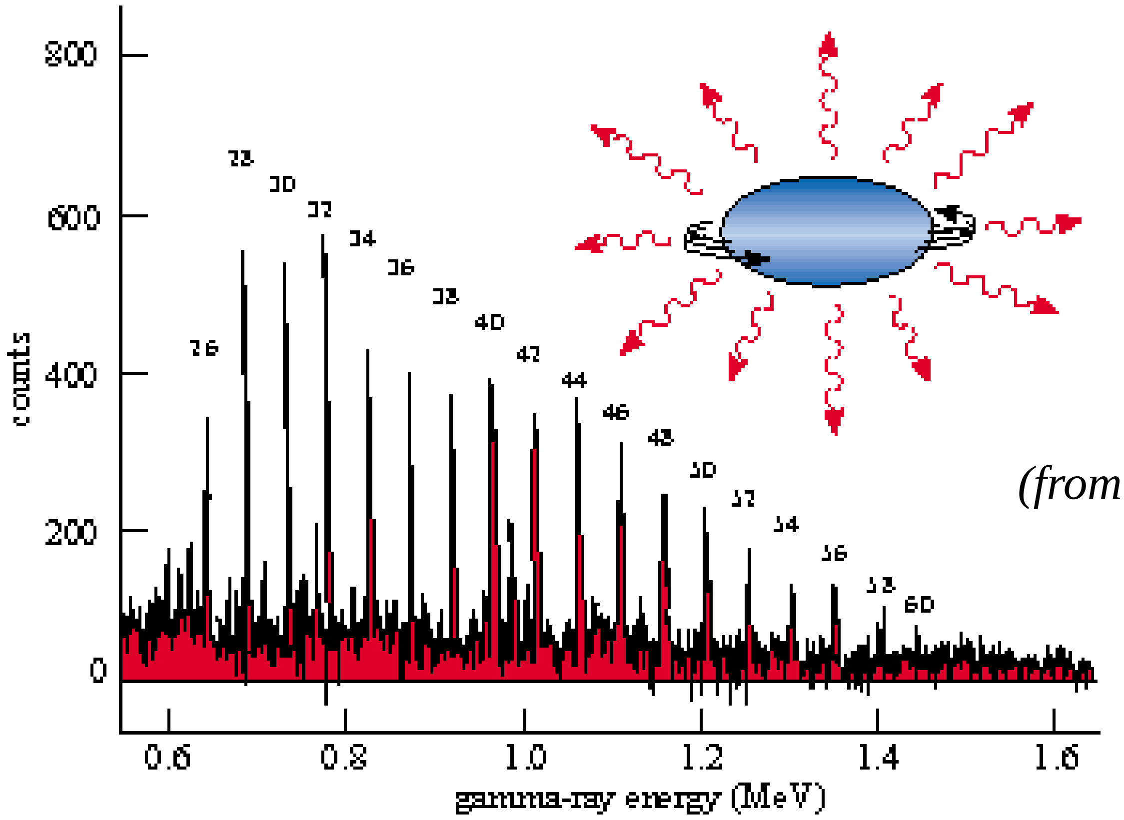


(from ref. 10)



4π detector "Gammasphere"
(Argonne National Laboratory, U.S.)

^{152}Dy : Emission of γ rays.

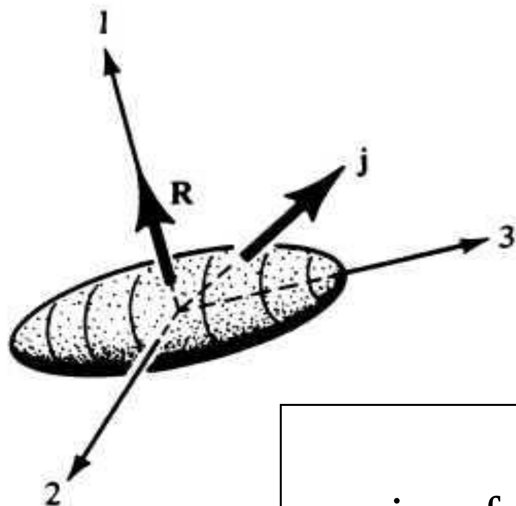


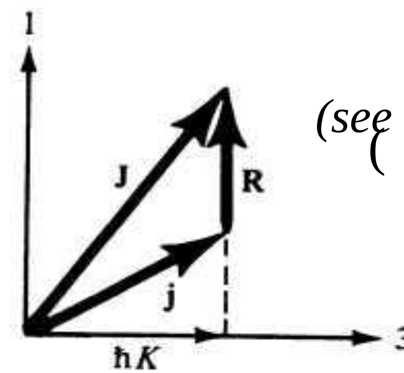
(from ref. 10)

Gamma-ray spectrum of a rotating dysprosium-152 nucleus. The numbers indicated are the angular momenta (in units of $\hbar$) which are taken away by the gamma-ray.

In the examples discussed so far, the ground state of the deformed nucleus has J=0. (Angular momentum arises due to collective rotation.)

Other example: Nucleus with a **deformed "core"** (which can rotate collectively with angular momentum **R**), **plus one valence nucleon** (angular momentum **j**), which moves in the potential of the deformed core:





(see ref. 11, sect. 18.3)

$K = J_3 = j_3$ is conserved, and arises from single particle motion in deformed field.
$R_3 = 0$, $J \geq K$

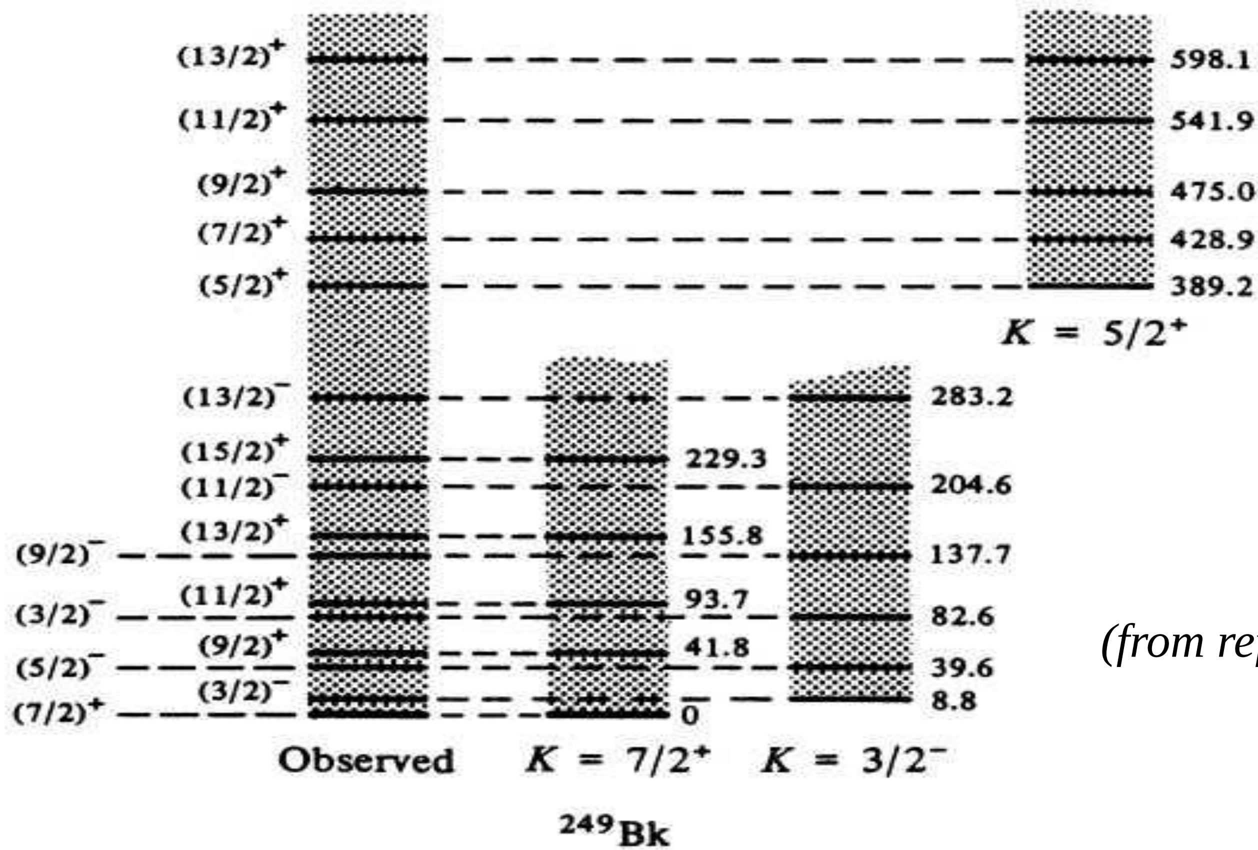
For each possible K ("**intrinsic state**"), there can be several collective rotations R ⟶ <u>**"Rotational bands"**</u>

The energies are then given by

$$E_{JK} = \frac{\hbar^2}{2I}\left[J(J+1) - 2K^2\right] + \varepsilon(K)$$

[J=K, K+1, K+2, …;
ε = single particle energy
(energy of "intrinsic state")]

Example: Spectrum of ^{249} Bk (Berkelium)



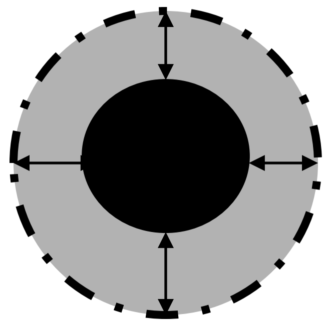
(from ref. 11)

iii) **Nuclear vibrations:**

Oscillations of nuclear shape around equilibrium.

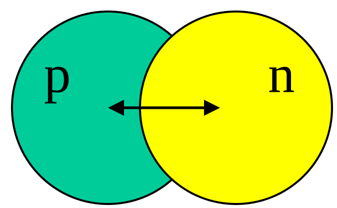
The most common types of nuclear vibrations are:

Monopole oscillations



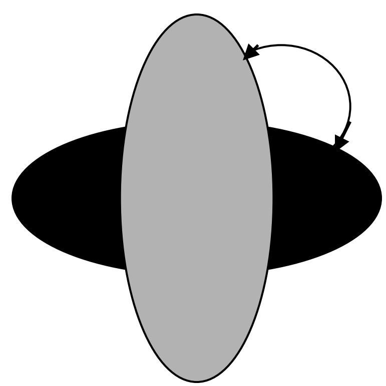
Compressional mode ("*breathing* mode")

Dipole oscillations



Oscillation of protons against neutrons

Quadrupole oscillations

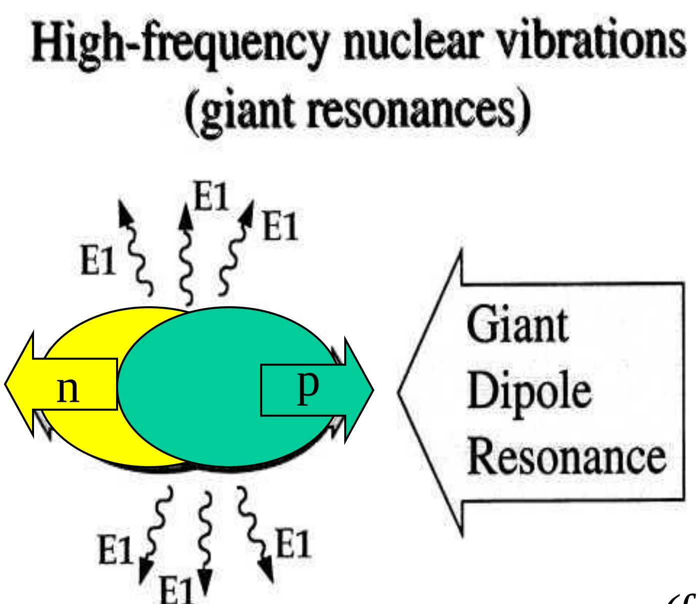


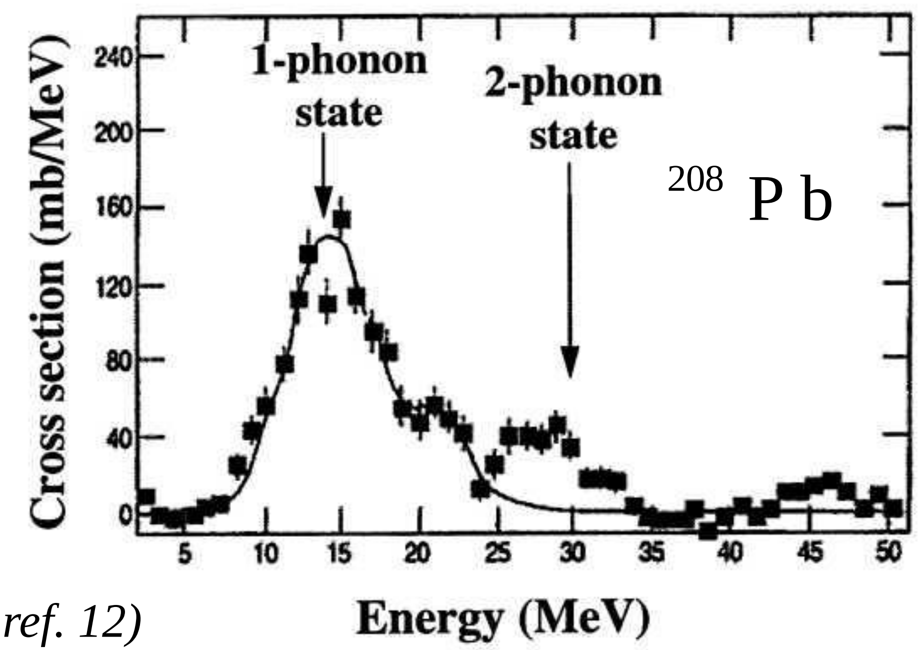
Shape deformations (volume conserved)

For high frequencies (excitation energies > 10 MeV) and large probabilities (cross sections) for excitation, these vibrations are called **"GIANT RESONANCES"**.

Giant resonances can be excited by bombarding nuclei with e^-, p, α, other nuclei, ...

The most prominent example is the **Giant Dipole Resonance (GDR)**, which has been found in most nuclei at excitation energy $\approx 77 \times A^{-1/3}$ MeV. For example, in ^{208}Pb :



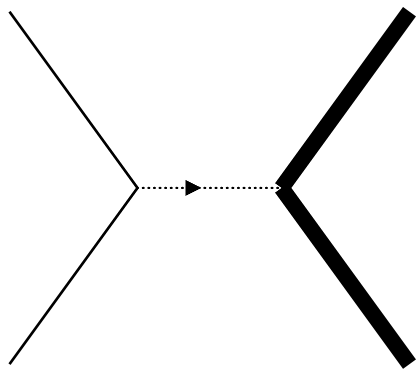


(from ref. 12)

How can we get this classical picture of oscillations from quantum mechanics?

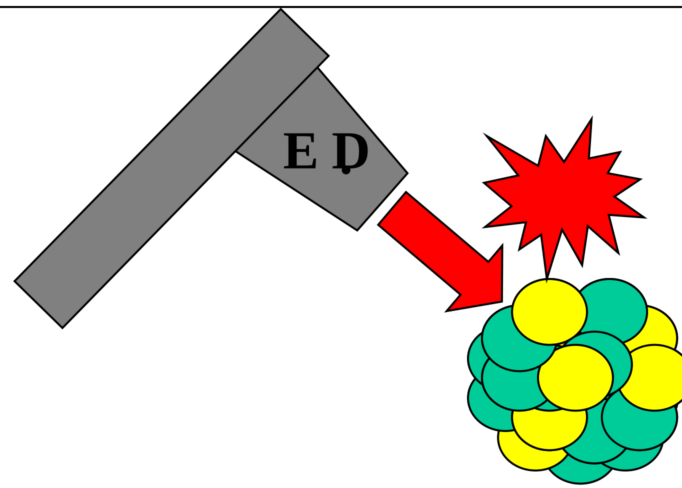
To see this, let us do some exercise in quantum mechanics…

Let us **hit the nucleus** by an external electric field **E**, and calculate the **induced current.**



electron nucleus

Electron-nucleus scattering



nucleus

Instantaneous perturbation

$V(t) = \mathbf{E}\cdot\mathbf{D}\,\delta(t)$

$\mathbf{D}$ = dipole operator = $\sum_{i=1}^{Z} \mathbf{r}_i$

Hamiltonian: $H = H_0 + V(t), \quad H_0 |n\rangle = E_n |n\rangle$

Use <u>time-dependent first order perturbation theory:</u>

$$
\begin{aligned}
|\psi\rangle(t) &= \sum_n c_n(t)|n\rangle e^{-iE_nt/\hbar} \\
c_n(t) &= \delta_{n0} + \frac{1}{i\hbar}\int_{-\infty}^{t}\langle n|V(t')|0\rangle e^{i\omega_n t'} \\
&= \delta_{n0} + \frac{1}{i\hbar}\langle n|\boldsymbol{E}\cdot\boldsymbol{D}|0\rangle
\end{aligned}
$$

Here $t > 0$, and $\omega_n = (E_n - E_0)/\hbar$.

<u>Induced current:</u>

$$\boldsymbol{j}(\boldsymbol{r},t) = \langle\psi(t)|\hat{\boldsymbol{j}}(\boldsymbol{r})|\psi(t)\rangle \equiv \frac{1}{i\hbar}(\boldsymbol{A} - \boldsymbol{A}^*)$$

where

$$\boldsymbol{A}(\boldsymbol{r},t) = \sum_n \langle 0|\hat{\boldsymbol{j}}(\boldsymbol{r})|n\rangle\langle n|\boldsymbol{E}\cdot\boldsymbol{D}|0\rangle e^{-i\omega_n t}$$

Using time reversal invariance, we get

$$\boldsymbol{j}(\boldsymbol{r},t) = \frac{2}{i\hbar}\sum_n \langle 0|\hat{\boldsymbol{j}}(\boldsymbol{r})|n\rangle\langle n|\boldsymbol{E}\cdot\boldsymbol{D}|0\rangle \cos\omega_n t \tag{1}$$

Define the giant resonance state $|R\rangle$ as the state which is excited most strongly by the operator $\boldsymbol{E} \cdot \boldsymbol{D}$, i.e.,

$$\boldsymbol{j}(\boldsymbol{r}, t) \simeq \frac{2}{i\hbar} \boldsymbol{j}_{\mathrm{tr}}(\boldsymbol{r})\, C_R \cos \omega_R t \qquad (2)$$

with the "transition current"

$$\boldsymbol{j}_{\mathrm{tr}}(\boldsymbol{r}) = \langle 0 | \hat{\boldsymbol{j}}(\boldsymbol{r}) | R \rangle$$

and

$$C_R = \langle R | \boldsymbol{E} \cdot \boldsymbol{D} | 0 \rangle \,, \qquad \omega_R = \frac{E_R - E_0}{\hbar}$$

On the other hand, using again time reversal invariance and completeness, we can calculate the sum in eq.(1) directly at $t = 0^+$:

$$\begin{aligned} \boldsymbol{j}(\boldsymbol{r}, 0^+) &= \frac{2}{i\hbar} \sum_n \langle 0 | \hat{\boldsymbol{j}}(\boldsymbol{r}) | n \rangle \langle n | \boldsymbol{E} \cdot \boldsymbol{D} | 0 \rangle \\ &= \frac{1}{i\hbar} \langle 0 | \left[\hat{\boldsymbol{j}}(\boldsymbol{r}), \boldsymbol{E} \cdot \boldsymbol{D} \right] | 0 \rangle \end{aligned}$$

Choose $\boldsymbol{E} = (E, 0, 0)$ and calculate the cummutator using the current operator

$$\hat{\boldsymbol{j}}(\boldsymbol{r}) = \frac{1}{2M} \sum_{i=1}^{Z} \left(\delta(\boldsymbol{r} - \boldsymbol{r}_i)\, \boldsymbol{p}_i + \boldsymbol{p}_i\, \delta(\boldsymbol{r} - \boldsymbol{r}_i)\right)$$

(Here $\boldsymbol{p}_i = -i\hbar \boldsymbol{\nabla}_i$.) Then we get

$$\boldsymbol{j}(\boldsymbol{r}, 0^+) = \frac{-1}{M} E\, \hat{\boldsymbol{x}}\, \rho(\boldsymbol{r})\,, \tag{3}$$

where $\hat{\boldsymbol{x}} = (1, 0, 0)$ is the unit vector in x-direction, and the charge density of the nuclear ground state is given by

$$\rho(\boldsymbol{r}) = \langle 0 | \sum_{i=1}^{Z} \delta(\boldsymbol{r} - \boldsymbol{r}_i) | 0 \rangle .$$

Therefore, using eqs.(2) and (3) we obtain finally

$$\boxed{\begin{aligned} \boldsymbol{j}(\boldsymbol{r}, t) &= \frac{-1}{M} E\, \hat{\boldsymbol{x}} \rho(\boldsymbol{r}) \cos \omega_R t \\ &= -\frac{E}{M} \sum_{i=1}^{Z} \langle 0 | \hat{\boldsymbol{x}}_i(t) \delta(\boldsymbol{r} - \boldsymbol{r}_i) | 0 \rangle \end{aligned}}$$

with the oscillating proton coordinates

$$\boxed{\boldsymbol{x}_i(t) = \boldsymbol{x}_i\, \cos \omega_R t}$$

We therefore see that the protons oscillate against the neutrons with the frequency

$$\omega_R = \frac{E_R - E_0}{\hbar} \simeq \frac{14\,\text{MeV}}{6.6 \times 10^{-22}\text{MeV} \cdot s} = 2.1 \times 10^{22} s^{-1}$$

Here we used $E_R = 14$ MeV for a heavy nucleus like ^{208}Pb. Note that this is a fast oscillation: The period $T_R = 2\pi/\omega_R \simeq 3 \times 10^{-22}s$ is about the same as the times T_{sp} it takes a nucleon on the Fermi surface to move across the nucleus and back:

$$T_{sp} = \frac{4R}{v_F} = \frac{4 \times 1.1 \times A^{1/3}\,\text{fm}}{0.3 \times (3 \times 10^{23}\,\text{fm}/s)} \simeq 3 \times 10^{-22}s$$

(Here $v_F \simeq c/3$ is the Fermi velocity.)

Note: "Time reversal invariance" in this derivation implies that

$$\begin{aligned} \langle n|\hat{\boldsymbol{j}}(\boldsymbol{r})|0\rangle &= -\langle 0|\hat{\boldsymbol{j}}(\boldsymbol{r})|\overline{n}\rangle \\ \langle n|\boldsymbol{E} \cdot \boldsymbol{D}|0\rangle &= \langle 0|\boldsymbol{E} \cdot \boldsymbol{D}|\overline{n}\rangle \end{aligned}$$

where $|\overline{n}\rangle$ is the time-reversed state of $|n\rangle$.

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